Estudios de Administración, vol. 12, Nº 2, 2005

Adjustment of the WACC with Subsidized Debt in the Presence of Corporate Taxes: The Finite-Horizon Case

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Abstract

When discounting free-cash flows (FCF) at the Weighted Average Cost of Capital (WACC), we assume that the cost of debt is the market, unsubsidized rate. With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain a loan at a rate that is below the market rate. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would differ. The benefit of lower tax savings are offset by the benefit of the subsidy. These two benefits have to be introduced explicitly.

Key Words: Adjusted present value, weighted average cost of capital, subsidized debt with taxes.
JEL Classification: D61, G30, G31, G32, H43

In this article, we present the necessary adjustments to the WACC and the cost of leveraged equity under the existence of subsidized debt and taxes in a multiple-period setting. We analyze the cases of the WACC applied to the FCF and the WACC applied to the capital-cash flows (CCF). We also utilize the Adjusted Present Value (APV) to consider both the tax savings and the subsidy. We show that all these different approaches give the same answer.

Extracto

A una tasa de interés de mercado, la deuda crea valor en la presencia de impuestos corporativos, al servir de escudo tributario. En algunos casos, no obstante, una empresa puede endeudarse por debajo del valor de mercado. En dicho escenario, el beneficio de un menor ahorro de impuestos se ve más que compensado por el beneficio del subsidio.

En este artículo, presentamos los ajustes necesarios para el costo de capital promedio ponderado (WACC)) y el costo del capital accionario apalancado, en la presencia de deuda subsidiada e impuestos y bajo un horizonte de múltiples períodos. Utilizamos, además, el método del valor presente ajustado (APV) para analizar el ahorro en impuestos y el beneficio de la deuda subsidiada. Demostramos que estos métodos alternativos de valorización entregan resultados coincidentes.

1. Introduction

When discounting free-cash flows (FCF) at the Weighted Average Cost of Capital (WACC), we assume that the cost of debt is the market, unsubsidized rate With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain

Palabras clave: Valor presente ajustado, costo capital promedio ponderado, deuda subsidiada e impuestos.
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a loan at a rate that is below the market rate. In previous work, we showed how to adjust the WACC in the presence of a subsidy and no taxes (Tham and Vélez-Pareja 2005). There we showed that plugging the lower cost of debt into the WACC formula is not the correct approach to measuring the value creation due to the subsidy. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would differ. The benefit of lower tax savings TS, is offset by the benefit of the subsidy. These two benefits have to be modeled explicitly.

In this article, we present the necessary adjustments to the WACC and the cost of leveraged equity under the existence of subsidized debt and taxes in a multiple-period setting. We analyze the cases of the WACC applied to the FCF and the WACC applied to the capital-cash flows (CCF). We also utilize the Adjusted Present Value (APV) to consider both the tax savings and the subsidy. We show that all these different approaches give the same answer.

The traditional WACC (expressed as $WACC_t = Kd_t \times D\%_{t-1} + Ke_t \times E\%_{t-1}$) is a very particular and special case of a more general formulation of WACC. However, this formulation is the most popular and it is used widely. This version of WACC is valid when some particular conditions are met, as follows:

- Taxes are paid the same period as accured.
- The only source of tax savings is the interest charges.
- There is enough profit (EBIT and/or other income) to offset the interest charges and hence, to earn in full the tax savings.

In the cases where subsidized debt is present, the use of this traditional expression requires additional adjustments. Our concern is to warn practitioners that eventually use the traditional WACC formulation (with the proper conditions met) in the sense that the effect of the subsidy has to be taken into account in an explicit way.

The proper cost of capital to discount the FCF varies depending on the market value of the firm and the value of TS. The market value of firms or projects not traded in the market is the present value of the cash flows discounted at the proper discount rate. For a proper discount rate, we understand the following (in the most general formulation)[1].

If we use the FCF, we should discount it at

$$WACC_{adjusted} = Ku_i - \frac{TS_i}{V^L_{i-1}} - (Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{V^L_{i-1}} \qquad (1)$$

where TS is the tax savings, ψ is the discount rate for the TS, Ku is the unlevered cost of equity, V is the total value and V^{TS} is the value of the future tax savings.

If we use the Capital Cash Flow CCF[2], (CFD + CFE = FCF + TS) we should discount it at

$$WACC_{for\ CCF} = Ku_i - \left(Ku_i - \psi_i\right)\frac{V^{TS}_{i-1}}{V^L_{i-1}} \qquad (2).$$

If we use CFE we should discount it with (in this case we have to add the value of debt to obtain the total value)

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}} \qquad (3),$$

where E is the market value of equity and D is the market value of debt and Kd is the cost of debt; other variables have been defined previously.

[1]Taggart (1991) presented the expressions for Ke and WACC and Vélez-Pareja and Tham (2000), Tham, and Velez–Pareja, 2002, Vélez-Pareja and Burbano, 2004, 2005 and Tham, Velez–Pareja, 2004a and 2004b, derived independently the expressions when cash flows are finite.

[2]The Capital Cash Flow (CCF) is the essence of the original Modigliani and Miller proposal in 1958; however, it was popularized by Ruback, 2000.

Of course we have to make explicit the assumption on ψ, the discount rate for the TS. We have assumed the ψ is the unlevered cost of equity, Ku.

When the discount rate for the tax savings ψ is Ku the previous expressions are

$$WACC_{for\ CCF} = Ku_i \tag{4a}$$

$$WACC_{adjusted} = Ku_i - \frac{TS_i}{V^L_{i-1}} \tag{4b}$$

and

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} \tag{5}$$

When the leverage is not constant and hence the Ke should change accordingly.

As we mentioned above, when there is a negative EBIT, the TS are not earned in the period. If there is the possibility of carried-forward losses, then we can recover the TS not earned during the loss period. This means that the traditional WACC cannot be used.

The issue of the effect of a subsidy in the interest rate on the WACC is not widely dealt with in the literature. Ross *et al*. (1999) mention its effect on the firm value and propose to use the APV method, while Damodaran 1996 suggests including the value of the subsidy in the cash flow. Dailami and Klein (1997) say "investors ask for government support in the form of grants, preferential tax treatment, debt or equity contributions, or guarantees" and that "Guarantees themselves do not appear to affect the cost of capital, which is determined by the risks of the project, not the financing structure". On the other hand, Krishnaswami and Subramaniam (2000) and Fratantoni and Niculescu (2005) discuss the effect of subsidy in interest on the acquisitions of households. Most literature studies the subsidy from the government due to the tax savings that arise from the corporate taxes. These references suggest that the real effect of subsidy in debt is not well incorporated in the cost of capital. While it is true that the proper way to deal with externalities, such as tax savings and subsidies, is the APV, many textbooks and practitioners still use the traditional WACC, and neglect the effect of the subsidy in the firm value. Most of them use the subsidized cost of debt in the formula[3]. For this reasons, we consider relevant to show how the subsidy should be incorporated into the most popular approach, that is, the traditional WACC approach.

In this article, we do properly incorporate the subsidy effect into the cost of capital. Moreover, we show that when improperly done, a lower cost of debt might destroy value instead of creating value.

This article is organized as follows. In Section 2, we present the expressions for the cost of capital in the presence of subsidy and corporate taxes for multiple periods, and we illustrate it with an example. Section 3 concludes. An appendix shows the derivation of the formulae used in Section 2.

[3]We conducted an informal survey by email among practitioners and finance teachers from Colombia, Chile and Argentina asking how they would take into account the fact of a subsidized cost of debt. The response rate was about 25% and we received 24 responses. The tally is as follows: 10 said to include the subsidized cost in the WACC, 6 suggested it should be included in the analysis in a different way, 5 did not know how to respond or not responded at all the specific question and 3 expressed other non conclusive opinions. This means that more than 50% of the respondents that gave a relevant answer suggested including the subsidized cost in the WACC.

2. Cost of capital in the presence of subsidy and corporate taxes

A summary of our results are shown in Table 1a.

Table 1a

Summary of formulae for different discount rates

CASH FLOW	DISCOUNT RATE
CFE	$\mathrm{Ke} = \mathrm{Ku} + \frac{\mathrm{D}}{\mathrm{E}}\left(\mathrm{Ku} - \mathrm{Kd}^{\mathrm{Sub}}\right) + \mathrm{V}_o^{L\,Sub}\frac{\lambda - \mathrm{Ku}}{\mathrm{E}} + \mathrm{V}_0^{\mathrm{TS}}\frac{\psi - \mathrm{Ku}}{\mathrm{E}}$
CCF	$\mathrm{WACC}^{\mathrm{CCF}} = \mathrm{Ku} + \frac{\mathrm{V}_0^{\mathrm{TS}}}{\mathrm{V}_0^{\mathrm{L}}}(\psi - \mathrm{Ku}) + \frac{\mathrm{V}_0^{L\,Sub}}{\mathrm{V}_0^{\mathrm{L}}}(\lambda - \mathrm{Ku})$
FCF	$\mathrm{WACC}^{\mathrm{FCF}} = \mathrm{Ku} + \frac{\mathrm{V}_0^{\mathrm{TS}}}{\mathrm{V}_0^{\mathrm{L}}}(\psi - \mathrm{Ku}) + \frac{\mathrm{V}_0^{L\,Sub}}{\mathrm{V}_0^{\mathrm{L}}}(\lambda - \mathrm{Ku}) - \frac{TS}{V_0^L} - \frac{\mathrm{V}_0^{\mathrm{L\,Sub}}(1+\lambda)}{V_0^L}$

Where Ke is the cost of leveraged equity and Ku is the cost of unleveraged equity, D is the market value of debt, E is the market value of equity, V^L is the leveraged value, let V^{Un} is the unleveraged value, V^{TS} is the value of the TS, V^{LSub} is the value of the interest subsidy, λ is the appropriate discount rate for the interest subsidy and ψ is the discount rate for the tax savings, TS.

These formulations are general and work for any D/E setup. In other words, the formulation for the WACC (even for the above mentioned traditional one) does not require constant target leverage. This fact generates what is known as circularity between values and discount rates: the value depends on the WACC and the WACC depends on value.

This problem can be easily solved: in a spreadsheet (Excel, for instance) select *Tools*, then select *Options*, and there, select the tab *Calculate* and tick the option *Iterations*. Done this, the formulas (4b) and (5) can be evaluated.

From this summary, we can obtain simpler formulations depending on the assumptions regarding the discount rate for TS and subsidy. For instance, if we assume that ψ and λ are equal to Ku, then the formulae for the different costs are shown in Table 1b:

Table 1b

Formulae assuming $\lambda = \psi = Ku$

CASH FLOW	DISCOUNT RATE
CFE	$Ke=Ku+(Ku-Kd^{Sub})D/E$
CCF	$WACC^{CCF}=Ku$
FCF	$WACC^{FCF}=Ku-TS/V^{L}_{0}-Sub/V^{L}_{0}$

The formula for Ke resembles the typical formulation of Ke when ψ is Ku, except that Kd is replaced by Kd^{Sub}. For the CCF we have $WACC^{CCF}$ equal to Ku; this is what is expected when we use the CCF and assume Ku as the discount rate for TS. Finally, for discounting the FCF we have $WACC^{FCF}$ equal to $Ku - TS/V^{L}_{0} - Sub/V^{L}_{0}$ and this resembles the adjusted WACC. (See Tham and Velez-Pareja 2004).

We illustrate these ideas with a three-period numerical example. The values of the various parameters are shown below. We present the input variables and the final tables after solving the circularity that arises when discounting the free-cash flows at the WACC to find the firm value[4].

As we mentioned in the Introduction, we show an example illustrating the use of the traditional WACC and how the subsidy should be included in the calculation. Again, the use of the traditional WACC formulation is widespread and it is used even when the basic required assumptions are not fulfilled.

The input variables are shown in Table 2.

[4]See Velez-Pareja and Tham (2001), Tham and Velez-Pareja (2004), and Velez-Pareja and Tham (2005).

Table 2

Input variables for single period example

Tax rate	20.0%
Cost of unleveraged equity, Ku	15.0%
Debt, D_0	842.67
Market cost of debt, Kd^{NS}	10.0%
FCF, constant	1,230.2
Subsidy on Kd	2.0%
Discount rate for Subsidy, λ	10.0%
Discount rate for TS, ψ	10.0%

Next we calculate the CFD with Kd^{Sub}, the TS, the subsidy and the CFE. These values will be needed to calculate Ke and WACC for FCF and CFE (Table 3a).

Table 3a

Kd^{Sub}, CFD, TS, Subsidy CFE, V^{TS}_0 and V^{LSub}_0

Year	0	1	2	3
Kdsub		8.0%	8.0%	8.0%
Value of debt	842.669	842.669	842.669	
CFD		67.4	67.4	910.1
TS		13.5	13.5	13.48271
Subsidy		16.9	16.9	16.9
FCF		1,230.2	1,230.2	1,230.2
CFE = FCF + TS + Sub - CFD		1,193.2	1,193.2	350.5
V^{TS}_0	33.5295	23.3997	12.2570	
V^{LSub}_0	41.9119	29.2497	15.3213	

Now we can calculate the value of Ke and the market value of equity for every year (Table 3b).

Table 3b
Leverage D% at market value, Ke and leveraged value of equity

YEAR	0	1	2	3
D%	29.22%	41.05%	76.79%	
D%/E%	0.413	0.696	3.309	
Ke		17.7044%	19.6575%	37.6201%
Leveraged equity value	2,041.670	1,209.980	254.676	
Leveraged value = Equity+debt	2,884.339	2,052.649	1,097.346	

For instance, for year 1, in the previous table we apply the equation

$$Ke=Ku+(D/E)(Ku-Kd^{Sub})+V^{LSub}{}_0(\lambda-Ku)/E+V^{TS}{}_0(\psi-Ku)/E$$

$$15\%+0.413\times(15\%-8\%)+41.9119\times(10\%-15\%)/2{,}041.670+33.5295\times(10\%-15\%)/2{,}041.670=17.7044\%$$

(allow for rounding errors if the reader tries to replicate this calculation).

Table 4 shows the computation of the present value by discounting the free-cash flow at WACC:

Table 4
FCF, WACC ^{FCF} and leveraged value

YEAR	0	1	2	3
FCF		1,230.2	1,230.2	1,230.2
$WACC^{FCF}$		13.82%	13.39%	12.11%
PV of FCF @ WACC	2,884.3393	2,052.6494	1,097.3457	

In the case of $WACC^{FCF}$ we have for year 1,

$$Ku+(V^{TS}{}_{0}/V^{L}{}_{0})(\psi-Ku)+V^{LSub}{}_{0}/V^{L}{}_{0}(\lambda-Ku))-TS/V^{L}{}_{0}-Sub/V^{L}{}_{0}$$

$$15\%+(33.5295/2{,}885.860)\times(10\%-15\%)+(43.4328/2{,}885.860)\times(8\%-15\%)$$
$$-13.5/2{,}885.860-16.9/2{,}885.860=13.79\%$$

Table 5 in turn reports the computation of the leveraged adjusted present value:

Table 5

Unleveraged values, values of TS and subsidy and APV

YEAR	0	1	2
Unleveraged value	2,808.8979	2,000.0000	1,069.7674
$V^{TS}{}_{0}$	33.5295	23.3997	12.2570
$V^{LSub}{}_{0}$	41.9119	29.2497	15.3213
Leveraged value APV	2,884.3393	2,052.6494	1,097.3457

The figures from this table are taken from previous tables, except for the unleveraged value, which is calculated as the present value of the FCF at Ku. (Table 6)

Table 6

Capital Cash Flow, CCF, WACC^{CCF} and leveraged value

YEAR	0	1	2	3
WACC for CCF		14.87%	14.87%	14.87%
CCF=FCF+TS+Sub		1,260.6	1,260.6	1,260.6
CCF=CFD+CFE		1,260.6	1,260.6	1,260.6
PV(CCF)	2,884.3393	2,052.6494	1,097.3457	

The CCF is derived from the data in Table 2. The WACC^{CCF} is derived using the next equation. For year 1 we have:

$Ku+(V^{TS}_0/V^L_0)(\psi-Ku)+V^{LSub}_0/V^L_0(\lambda-Ku)$

$15\%+(33.5295/2{,}884.3393)\times(10\%-15\%)+(41.9119/2{,}884.3393)\times(10\%-15\%)=14.87\%$

Now we calculate the leveraged value assuming what the current practice is, i.e., to include the Kd^{Sub} in the traditional formula for WACC. We first calculate the leveraged value without subsidy. This is what is shown in Table 7.

Table 7

Computation of value using Kd^{NS} and FCF

YEAR	0	1	2	3
Market cost of debt, Kd^{NS}		10.0%	10.0%	10.0%
Debt (% of leveraged value)	29.59%	41.56%	77.71%	
Debt-equity ratio	0.420	0.711	3.486	
Ke		17.10%	18.56%	32.43%
WACC		14.4%	14.2%	13.4%
FCF		1,230.2	1,230.2	1,230.2
Leveraged value	2,847.38	2,027.40	1,084.42	

Now we calculate the value using the traditional WACC for the FCF and including Kd^{Sub} as the cost of debt (Table 8).

Notice that the leveraged value decreases as compared with the case where we use the traditional WACC and use the Kd^{Sub}. A lower cost of debt destroys value! This is counter-intuitive. This occurs because we have disregarded part of the value generated by the TS, and because the Ke calculation absorbs the reduction of the debt cost. This means that the subsidy has to be explicitly included into the analysis.

Table 8

Computation of leveraged value using Kd^{Sub} and FCF

YEAR	0	1	2	3
Kd^{Sub}		8.0%	8.0%	8.0%
Debt (% of leveraged value)	29.67%	41.68%	77.92%	
Debt-equity ratio	0.422	0.715	3.528	
Ke		17.95%	20.00%	39.70%
WACC		14.53%	14.33%	13.75%
FCF		1,230.2	1,230.2	1,230.2
Leveraged value	2,839.68	2,021.92	1,081.49	

In Table 9, we present a summary of the different computations.

Table 9

Different values with different methods

METHOD	LEVERAGED VALUE	EQUITY VALUE
No subsidy	2,847.4	1,997.03
With subsidy using Kd^{Sub} in the WACC	2,839.7	2,004.73
With subsidy using new formulation for WACC	2,884.34	2,041.67

In the numerical example, we assume that the appropriate discount rate for the interest subsidy λ is the subsidized rate of interest. However, we could also use the market rate Kd or the Ku. For completeness, in the next table we show the consistent results for the two other values for λ, namely Kd^{Sub} and Ku.

It might be argued that the differences in this example are irrelevant (Table 10). However, we think that it is not a matter of precision; it is a matter of correctness that can be reached without an extra cost. Moreover, it is customary to assume that the differences arise due to rounding errors, or that the magnitude is negligible or that practical approaches are more important than theoretical and precise ones. However, while errors could cancel out, sometimes errors add up. See for instance Vélez Pareja 2004 and 2005.

Table 10

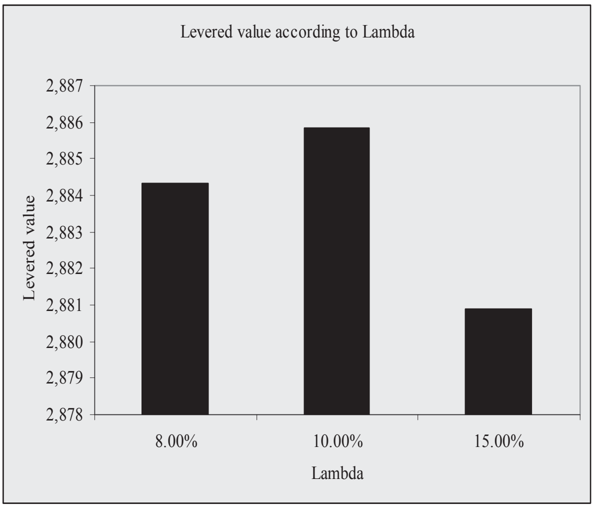
Results for different values of λ

	UNSUBSIDIZED	SUBSIDIZED	KU
λ	10.00%	8.00%	15.00%
Equity, unsubsidized debt	1,997.01	1,997.01	1,997.01
Equity, subsidized debt	2,041.67	2,043.19	2,038.24
Levered value, unsubsidized debt	2,847.38	2,847.38	2,847.38
Levered value, subsidized debt	2,839.68	2,839.68	2,839.68
Value, using APV, WACC for FCF, CFE with Ke and WACC for CCF	2,884.34	2,885.86	2,880.91

In Figure 1 we show the same results graphically:

Figure 1

Values for different levels of λ the discount rate of the subsidy



3. Conclusion

In this paper, we show the adjustments that have to be made to the WACC in the presence of a subsidized loan and taxes. It is interesting to observe that under the existence of a debt subsidy, plugging such lower debt cost into the WACC is not the correct approach to measure the firm value increase due to the subsidy. The adjustments to the WACC and the explicit introduction of the subsidy into the analysis provide the right answer.

We found that the discount rate for the subsidy affects the value of the firm. As expected, when λ the discount rate of the subsidy equals Ku, the firm value is lower. However, the use of Kd as discount rate for the subsidy does not result in a lower value. Instead, it yields the highest value.

As can be noticed there is consistency between all the values calculated with these different methods. This consistency is attained using the proper formulation the cost of levered equity and WACC, and solving the circular relationship that arises when we calculate firm value and cost of capital. These findings and more details on the procedure can be found in Vélez-Pareja, Ignacio and Joseph Tham (2000, 2005) and Tham and Vélez-Pareja (2004).

Appendix

1. Proper Formulations for Ke and WACC in a Finite Horizon Case for Any Number of Periods.

In this appendix we derive the proper formulations for Ke and WACC for any t.

First we derive the cost of leveraged equity, Ke. Let V^L be the leveraged value, let V^{Un} be the unleveraged value, let V^{TS} the value of the TS, let T the corporate tax rate and let V^{LSub} be the value of the interest subsidy. Then, with respect to the end of any t, the leveraged value equals the sum of the unleveraged value, plus the value of the TS and the value of the interest subsidy.

$$V^L{}_t = V^{Un}{}_t + V^{TS}{}_t + V^{LSub}{}_t \qquad (1)$$

This is a value conservation expression (there is an equivalent one for cash flows) extended from the one proposed by Modigliani and Miller, 1958, 1963 for perfect markets.

Using the APV approach, it would be very easy to estimate the value of the subsidized debt. Let Kd^{NS} be the cost of the non subsidized debt, and let Kd^{Sub} be the cost of the subsidized debt. The value of the debt at the end of year t is D_t. Let $L^{Sub}{}_t$ be the interest subsidy at the end of any year t and TS_t be the TS at the end of year t. Then the interest subsidy equals the value of the debt times the difference between the two interest rates adjusted for taxes and the TS are the cost of unsubsidized debt times the debt, D_0 and times the tax rate, T.

$$L^{Sub}{}_t = D_{t-1}(Kd^{NS} - Kd^{Sub}) \qquad (2)$$

and

$$TS_t = Kd^{Sub} \times T \times D_{t-1} \qquad (3).$$

The expression for the value of the interest subsidy is as follows, where λ is the appropriate discount rate for the interest subsidy.

$$V^{LSub}_{t-1}=L^{Sub}_{t}/(1+\lambda)=D_{t-1}(Kd^{NS}-Kd^{Sub})/(1+\lambda) \qquad (4).$$

The expression for the value of the TS is as follows, where ψ is the appropriate discount rate for the TS.

$$V^{TS}_{t-1}=Kd^{Sub}\times T\times D_{t-1}/(1+\psi)=D_{t-1}\times T\times Kd^{Sub}/(1+\psi) \qquad (5)$$

where ψ is the discount rate for the tax savings, TS.

2. Derivation of Ke

Let CCF_t be the capital cash flow at the end of any year t with financing. At the end of year t, the capital cash flow equals the sum of the FCF, plus the TS and the interest subsidy.
Then,

$$CCF_t=FCF_t+L^{Sub}_t+TS_t \qquad (6).$$

Also, at the end of year t, the capital cash flow equals the sum of the cash flow to equity (CFE) and the cash flow to debt (with the subsidized interest rate).

$$CCF_t=CFE_t+CFD_t \qquad (7).$$

Putting these two equations together, we obtain,

$$CCF_t = CFE_t + CFD_t = FCF_t + L^{Sub}_t + TS_t \qquad (8).$$

The corresponding value relationship is as follows.

$$V^{L}_{t-1} = E_{t-1} + D_{t-1} = V^{Un}_{t-1} + V^{LSub}_{t-1} + V^{TS}_{t-1} \qquad (9).$$

Substituting the appropriate value expressions for each of the cash flow items in equation 8, we obtain,

$$E_{t-1}\times(1+Ke)+D_{t-1}\times(1+Kd^{Sub}) =V^{Un}_{t-1}\times(1+Ku)+V^{LSub}_{t-1}\times(1+\lambda)+V^{TS}_{t-1}(1+\psi) \qquad (10)$$

where Ke is the cost of leveraged equity and Ku is the cost of unleveraged equity.

Applying equation 9 to equation 10, we obtain,

$$E_{t-1}\times Ke+D_{t-1}\times Kd^{Sub} = V^{Un}_{t-1}\times Ku + V^{LSub}_{t-1}\times\lambda + V^{TS}_{t-1}\times\psi \qquad (10.1)$$

$$E_{t-1}\times Ke+D_{t-1}\times Kd^{Sub}=(E_{t-1}+D_{t-1}-V^{LSub}_{t-1}-V^{TS}_{t-1})\times Ku+ V^{LSub}_{t-1}\times\lambda+V^{TS}_{t-1}\times\psi \qquad (10.2).$$

Rearranging, we obtain,

$$E_{t-1}\times Ke=E_{t-1}\times Ku+D_{t-1}\times(Ku-Kd^{Sub})+V^{LSub}_{t-1}\times(\lambda-Ku)+ V^{TS}_{t-1}\times(\psi-Ku) \qquad (11).$$

Substituting equation 4 and 5 into equation 11, we obtain the expression for the Ke.

$$E_{t-1}\times Ke=E_{t-1}\times Ku+D_{t-1}(Ku-Kd^{Sub})+D_{t-1}(Kd^{NS}-Kd^{Sub})(\lambda-Ku) /(1+\lambda)+[D_{t-1}\times T\times Kd^{Sub}/(1+\psi)](\psi-Ku) \qquad (12.1)$$

$$Ke=Ku+(Ku-Kd^{Sub})D_{t-1}/E_{t-1}+(Kd^{NS}-Kd^{Sub})[(\lambda-Ku)/(1+\lambda)] D_{t-1}/E_{t-1}+[T\times Kd^{NS}/(1+\psi)](\psi-Ku)D_{t-1}/E_{t-1} \qquad (12.2),$$

but
from (4) $V^{LSub}_{t-1}= D_{t-1}(Kd^{NS}-Kd^{Sub})/(1+\lambda)$ and
from (5) $V^{TS}_{t-1}=Kd^{Sub}\times T\times D_{t-1}/(1+\psi)=D_{t-1}\times T\times Kd^{Sub}/(1+\psi)$
then

$$Ke=Ku+(D/E)(Ku-Kd^{Sub})+V^{LSub}_{t-1}(\lambda-Ku)/E+V^{TS}_{t-1} (\psi-Ku)/E \qquad (12.3).$$

If we assume that the appropriate discount rate for the interest subsidy and for the TS is equal to the cost of unleveraged equity, then the third and fourth terms in equation 12.2 are zero.

3. Derivation of $WACC^{CCF}$

We now derive the WACC for the capital cash flow, CCF. From (8) we can write the following

$$V^{L}_{t-1}\times(t+WACC^{CCF}) = CCF_t = FCF_t + LSub_t + TS_t \quad (13)$$

and

$$V^{L}_{t-1}\times(1+WACC^{CCF}) = CCF_t = V^{Un}_{t-1}(1+Ku)+V^{LSub}_{t-1}\times(1+\lambda)+V^{TS}_{t-1}\times(1+\psi) \quad (14a).$$

As per (9) then

$$V^{L}_{t-1}\times WACC^{CCF} = V^{Un}_{t-1}\times Ku + V^{LSub}_{t-1}\times\lambda + V^{TS}_{t-1}\times\psi \quad (14b)$$

and

$$V^{L}_{t-1}\times WACC^{CCF} = (V^{L}_{t-1}-V^{LSub}_{t-1}-V^{TS}_{t-1})\times Ku + V^{LSub}_{t-1}\times\lambda + V^{TS}_{t-1}\times\psi \quad (14c).$$

Rearranging terms

$$V^{L}_{t-1}\times WACC^{CCF} = V^{L}_{t-1}\times Ku + V^{LSub}_{t-1}\times(\lambda-Ku) + V^{TS}_{t-1}\times(\psi-Ku) \quad (14d).$$

Dividing by V^{L}_{0}

$$WACC^{CCF} = Ku + (V^{TS}_{t-1}/V^{L}_{t-1})(\psi-Ku) + (V^{LSub}_{t-1}/V^{L}_{t-1})(\lambda-Ku) \quad (14e).$$

4. Derivation of $WACC^{FCF}$

Now we derive the WACC to be applied to the FCF. As before, from (8) we can write the following

$$V^{L}_{t-1}\times(t+WACC^{FCF}) + LSub_t + TS_t = FCF_t = CFE_t+CFD_t=CCF_t \qquad (15a)$$

$$V^{L}_{t-1}\times(1+WACC^{FCF}) + V^{LSub}_{t-1}\times(1+\lambda) + V^{TS}_{t-1}\times(1+\psi) = V^{L}_{t-1}\times(1+WACC^{CCF}) \qquad (15b).$$

Replacing the expression for $WACC^{CCF}$ we have

$$V^{L}_{t-1}\times(1+WACC^{FCF}) + V^{LSub}_{t-1}\times(1+\lambda) + V^{TS}_{t-1}\times(1+\psi) = V^{L}_{t-1}\times(1+Ku + (V^{TS}_{t-1}/V^{L}_{t-1})(\psi-Ku) + V^{TS}_{t-1}/V^{L}_{t-1}(\lambda-Ku)) \qquad (15c)$$

but

$$V^{LSub}_{t-1}\times(1+\lambda)=Sub \text{ and}$$

$$V^{TS}_{t-1}\times(1+\psi)=TS$$

Then

$$V^{L}_{t-1}\times(WACC^{FCF}) + Sub + TS = V^{L}_{t-1}\times(Ku + (V^{TS}_{t-1}/V^{L}_{t-1})(\psi-Ku) + V^{LSub}_{t-1}/V^{L}_{t-1}(\lambda-Ku)) \qquad (15d).$$

Dividing by V^{L}_{t-1}

$$WACC^{FCF} + Sub/V^{L}_{t-1} + TS/V^{L}_{t-1} = Ku + (V^{TS}_{t-1}/V^{L}_{t-1})(\psi-Ku) + V^{LSub}_{t-1}/V^{L}_{t-1}(\lambda-Ku)) \qquad (15e)$$

Rearranging terms

$$WACC^{FCF}=Ku+(V^{TS}_{t-1}/V^{L}_{t-1})(\psi-Ku)+V^{LSub}_{t-1}/V^{L}_{t-1}(\lambda-Ku))-TS/V^{L}_{t-1}-Sub/V^{L}_{t-1} \qquad (15f).$$

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